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Broadband Plasma Impedance Measurements and Determination of Plasma Parameters

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Broadband Plasma Impedance Measurements and Determination of Plasma Parameters

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A small spherical probe is used in conjunction with a network analyzer to determine the impedance of the probe-plasma system over a wide frequency range. Impedance curves are in good agreement with accepted circuit models with plasma-sheath and electron plasma frequency resonances easily identifiable. Phase measurements show clear transitions between capacitive and inductive modes as predicted by the model. Sheath thickness and absolute electron density are determined from the location of these transitions. In addition, much larger power absorption is observed at the sheath plasma resonance than is predicted by collisional absorption.

I. INTRODUCTION

The impedance of a probe in a plasma as a means of obtaining information about the plasma's characteristics is an attractive diagnostic for use in situations where more conventional Langmuir probes and charge collectors are inadequate. In early experiments by Harp and Crawford [1] investigating this technique, the Q of a spherical probe immersed in a plasma was used to estimate the plasma-sheath resonance frequency. Harp explained that a caveat of this measurement technique was that the impedance of the cable connected to the probe has to be small compared to the impedance of the sphere. In addition, there was no experimental data given regarding the plasma frequency measurement, the resonance at which was described as being difficult to define. Subsequent authors [2], [3] have performed experiments where the sheath plasma resonance was observed by changing the plasma density while keeping the frequency of the signal applied to the probe constant. Dote [4] measured the resonant frequencies of planar and spherical probes in a magnetized plasma by measuring the DC component of the probe current while sweeping the frequency. More recently, plasma-sheath resonances have been observed in a capacitively coupled discharge [5].

Quantitative measurements of the impedance itself are more rare to find in the literature because of the difficulties associated with eliminating stray capacitances and impedances in the probe circuit. The best examples can be found in sounding rocket experiments on ionospheric plasmas. Oya [6] included the capacitance of the metallic boom connected to a spherical probe, compensating for stray capacitance using a bridge circuit. Over the frequency range of the experiment, 0.5-16 MHz, the boom could be considered electrically short enough so that its inductance could be neglected. Jensen [7] measured changes in electron density using a phase locked loop circuit to track the frequencies at which a small antenna immersed in a plasma had an impedance that was large and nonreactive. More recently Steigies *et. al.* [8] used a capacitive bridge type circuit with a strip antenna. However, all

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of these experiments were performed in low density plasmas such that $\omega_{pe}/2\pi$ is less than 20 MHz. They also required specially designed electronics to overcome circuit effects. In this paper we extend the utility of this diagnostic method to much higher frequencies (1 GHz) corresponding to higher density plasmas. Impedance measurements are made with an off-the-shelf network analyzer; no custom parts or electronics are needed except for the spherical probe itself. Following the calibration procedure outlined in the owners manual of the network analyzer is all that is required to make use of this diagnostic.

II. THEORY AND EXPERIMENTAL TECHNIQUE

A. Theoretical Impedance

A spherical capacitor of radius R surrounded by a uniform, infinite plasma, biased at the plasma potential V_p , will have a capacitance given by

$$C = 4\pi\epsilon_0\epsilon_r R \quad (1)$$

with

$$\epsilon_r = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu)} \quad (2)$$

where ν is the total electron collision frequency. If the sphere is at a potential different from V_p , a sheath will form between its surface and the plasma. The overwhelming majority of the sheath impedance is capacitive, so the total impedance of the probe and plasma can be written as if it were a capacitance in series with a tank circuit, as shown in figure 1. The circuit elements are given by a sheath capacitance C_{sh} , the vacuum capacitance $C_0 = 4\pi\epsilon_0 R$, an inductor $L_p = \omega_p^{-2} C_0^{-1}$, and a resistance $R = \nu L_p$, with the impedance given by

$$Z = \frac{1}{j\omega C_{sh}} + \frac{1}{j\omega C_0 + \frac{1}{R_p + j\omega L_p}} \quad (3)$$

The sheath capacitance represents the capacitance of the region between the probe surface

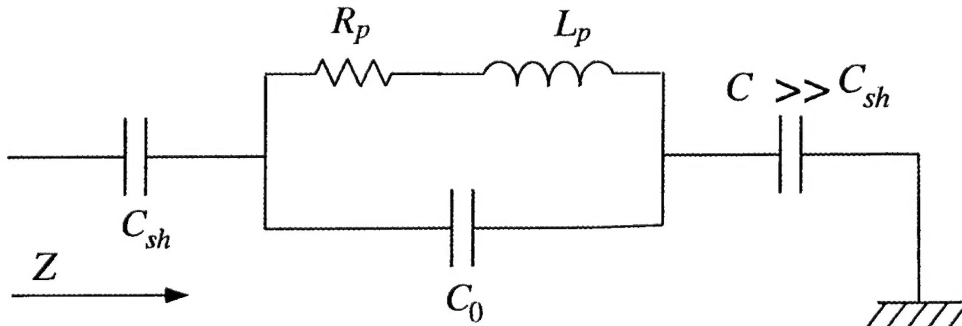


FIG. 1: Circuit model of the probe plasma impedance.

and the plasma, while the tank circuit represents the impedance between the sheath edge at the probe to the sheath edge at the chamber wall. Implied in this model is that the

plasma is uniform over a scale length much larger than the probe radius. This circuit model is identical to that outlined in Lieberman [11] for a parallel plate capacitive discharge; the only modifications are that a spherical geometry is used and the sheath capacitance C_s , representing the sheath between the vacuum vessel and the plasma, is neglected, owing to the much larger area of the space chamber with respect to the probe surface area.

Equation 3 can be rearranged using the definitions of R_p and L_p and defining $\gamma = \omega/\omega_p$ and $\delta = \nu/\omega_p$ to write

$$Re(Z) = \frac{1}{\omega_p C_0} \frac{\delta}{(\gamma^2 - 1)^2 + \gamma^2 \delta^2} \quad (4)$$

$$Im(Z) = -\frac{C_0 [(\gamma^2 - 1)^2 + \gamma^2 \delta^2] + C_{sh} [(\gamma^2 - 1)\gamma^2 + \gamma^2 \delta^2]}{\omega_p C_0 C_{sh} [(\gamma^2 - 1)^2 + \gamma^2 \delta^2]} \quad (5)$$

The magnitude and phase derived from 4 and 5 are plotted in figure 2 for $\delta = 0.01$. We can see that there are two frequencies of interest, γ_1 and γ_2 , where the phase of the impedance changes from $-\pi/2$ to $\pi/2$, corresponding to a null and a peak in the magnitude. For small δ , we can solve for $\gamma_{1,2}$ to find the corresponding points in frequency space as

$$\omega_1 \approx \omega_p \sqrt{\frac{C_0}{C_{sh} + C_0}} \quad (6)$$

$$\omega_2 = \omega_p \quad (7)$$

If we assume that the sheath is a vacuum region between the surface of the probe and the plasma with thickness s then $\omega_1 \approx \frac{\omega_p}{\sqrt{1 + \frac{R+s}{s}}}$

B. Measurement of Impedance

The complex impedance Z is obtained by using a HP8735D network analyzer to measure the reflection coefficient Γ , given by

$$\Gamma = \frac{Z - Z_0}{Z + Z_0} \quad (8)$$

with Z_0 the internal impedance of the analyzer, which in this experiment is 50Ω . Typically the sphere will be connected to a probe mount with 50Ω coax connected between it and the network analyzer. The network analyzer will see a terminal impedance of this entire assembly, not just the sphere at the end. The extra impedance from the cable and probe shaft can be compensated for calibrating the analyzer to ignore everything up to the very tip where the sphere is attached. The probe should therefore be constructed such that the sphere is removable and a short or 50Ω load can be put in its place with no additional cabling. Figure 3 shows how the probe tip is constructed from $1/4$ " semi-rigid coax with the outer jacket and dielectric removed to expose a short length at the tip. One at a time, the operator indicates that there is an open circuit ($\Gamma = 1$), a short circuit ($\Gamma = -1$), or a 50Ω load. The internal computer then adjusts internal phase and amplitude filters until these values of Γ are visible at the terminal. Figure 4 shows the measured phase and amplitude obtained as well as the expected theoretical values for the 19 mm diameter sphere used.

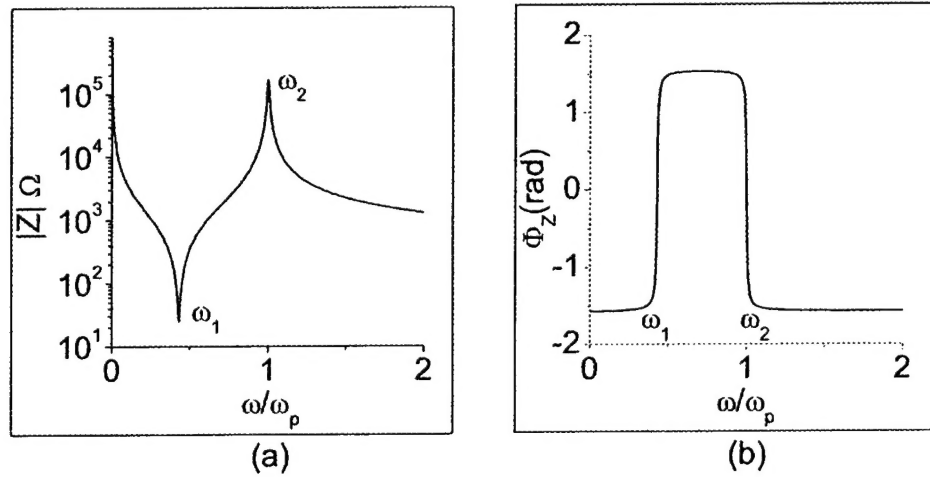


FIG. 2: Magnitude (a) and phase (b) of the impedance of a 19 mm diameter metal sphere immersed in a cold ($T_e = 0.5\text{eV}$) unmagnetized plasma with $\nu/\omega_p = 0.01$ and a sheath width of $5\lambda_D$.

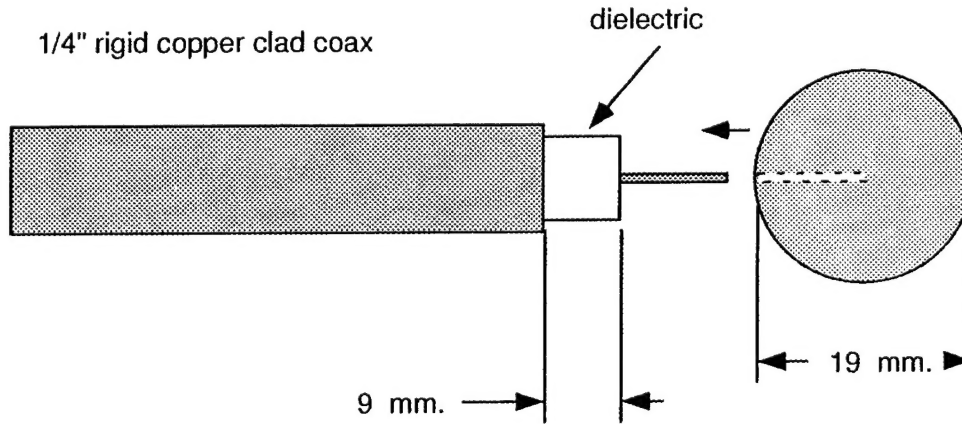


FIG. 3: Illustration of the spherical RF probe. The calibration is done by removing the sphere and attaching a $50\ \Omega$ load or short to the exposed tip with the return lead as short as possible and attached to the outer jacket of the coax. An open circuit is calibrated with nothing being attached; this eliminates almost all of the contribution from the capacitance between the sphere and the jacket.

C. Experimental Setup

The Space Physics Simulation Chamber at NRL has been described in several earlier experiments [9]. A very large vacuum vessel ($\approx 2\text{-m}$ diameter by 5-m length) is filled to a pressure of $p \approx 10^{-4}$ Torr Argon and surrounded by a weak magnetic field ($B_0 \approx 3$ gauss) such that $\Omega_e \ll \omega, \omega_{pe}$. A uniform, weakly ionized ($n \approx 10^7 - 10^9$) plasma is created with a large array of glowing tungsten filaments biased at -100 V. The electron and ion temperatures are typically $T_e \approx 0.5$ eV and $T_i \approx 0.05$ eV. The electron density is measured with a heated Langmuir probe mounted in proximity to the sphere. A drawing of the experiment is shown

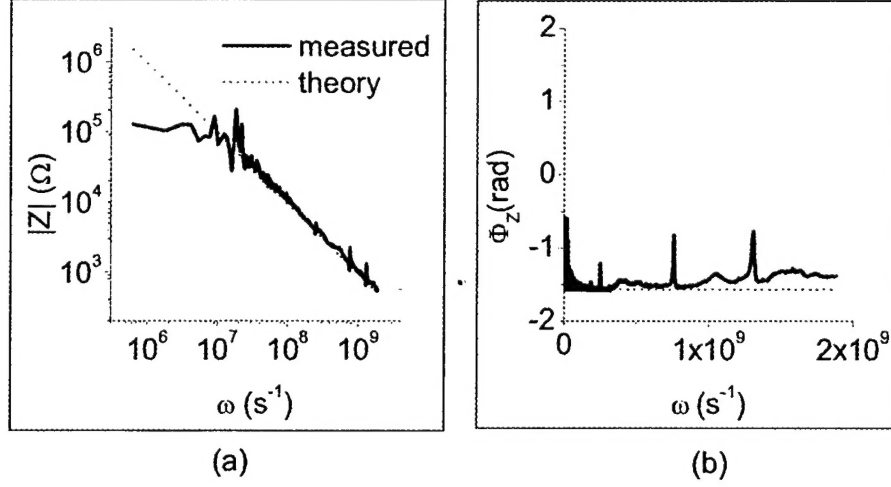


FIG. 4: Measured magnitude (a) and phase (b) of the impedance of the RF probe in vacuum compared with the calculated values for a 19 mm metal sphere.

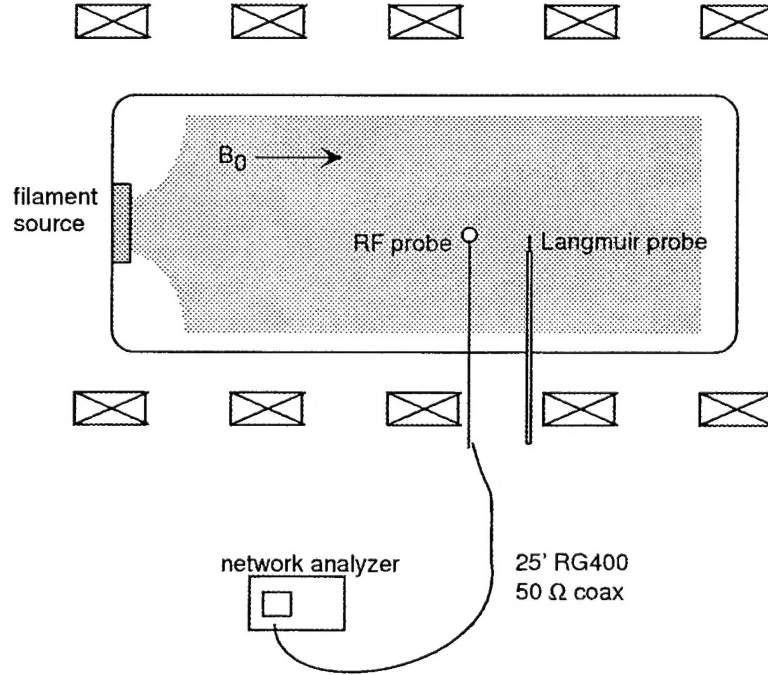


FIG. 5: Illustration of experimental setup in the SPSC plasma.

in figure 5.

III. RESULTS

Figure 6 shows the amplitude and phase of the probe impedance in a typical plasma with $n_e = 10^8 \text{ cm}^{-3}$ and a pressure of 3.4×10^{-4} Torr. For comparison the theoretical calculations

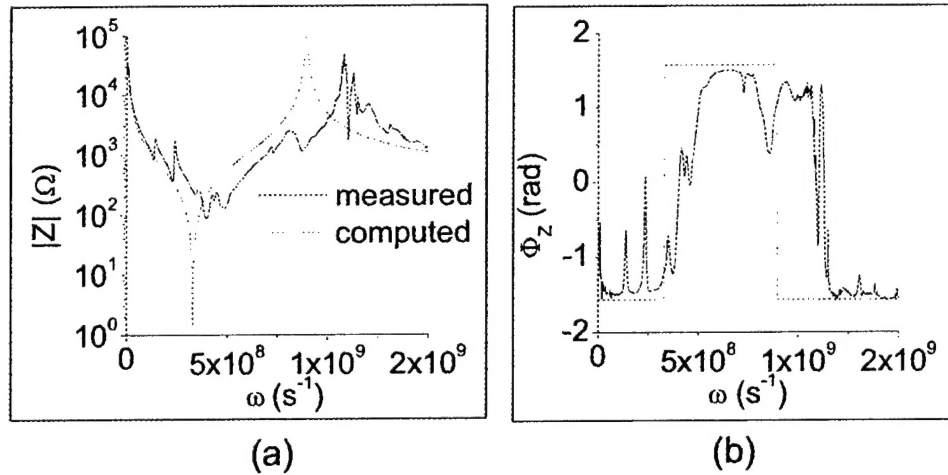


FIG. 6: Measured magnitude (a) and phase (b) of the impedance of RF probe immersed in a plasma with $n_e = 2.5 \times 10^8 \text{ cm}^{-3}$ and $T_e = 0.61 \text{ eV}$. The dashed lines are computed impedance curves from equation 3.

are also shown. There is a slight shift in frequency most likely due to error in estimating the collecting area. The resonant frequency ω_2 is plotted versus electron density inferred from the Langmuir probe in figure 7. Also shown is the plasma frequency derived from the formula $\omega_{pe}/2\pi = 8.98 \times 10^3 \sqrt{n_e}$. The plasma frequency calculated from Langmuir probe is an average of 15% lower than that of the spherical RF probe but does follow the correct trend.

Figure 8 shows the null frequency ω_1 versus the resonance frequency ω_2 as the electron density is increased. For comparison we also show $\omega_1 = \frac{\omega_p}{\sqrt{1 + \frac{R+s}{s}}}$. The sheath thickness s is assumed to be some number κ Debye lengths λ_D . The average value of s calculated from ω_1/ω_2 gives a value of $\kappa = 4.5$, which is on the order of that estimated by Harp and Crawford [1]. When a much larger sphere ($R = 7.6 \text{ cm}$) was substituted for the smaller sphere, the null frequency ω_1 became so small that the resolution $\Delta\omega$ was a significant fraction of the measurement. This is most likely the reason for larger errors seen in the plot (figure 9(b)) of ω_1 versus ω_2 corresponding to figure 8. Figure 9(a) shows the resonance frequency ω_2 is still a good approximation of the plasma frequency. An interesting feature of the impedance probe is that the reflection coefficient Γ also has a dip at $\omega = \omega_1$, indicating a large deposition of RF power (figure 10). This dip is what is commonly referred to as the sheath-plasma resonance. The theoretical curve corresponding to the plasma conditions is shown on the second y-axis. Although the location of the resonance is the same, the experimental dip is much deeper and broader than what theory predicts, requiring at least an order of magnitude increase in our estimate of the collision frequency.

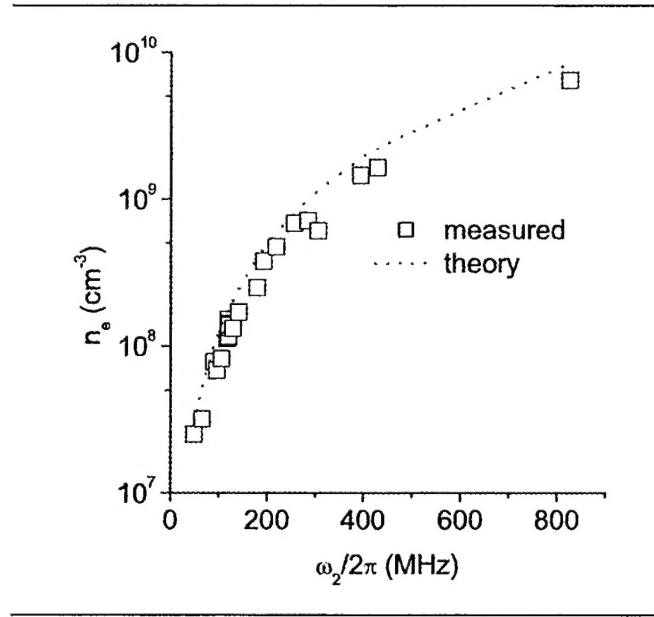


FIG. 7: Electron density vs. impedance resonance frequency. The dashed line is the theoretical value calculated from the formula $\omega_{pe}/2\pi = 8.98 \times 10^3 \sqrt{n_e}$

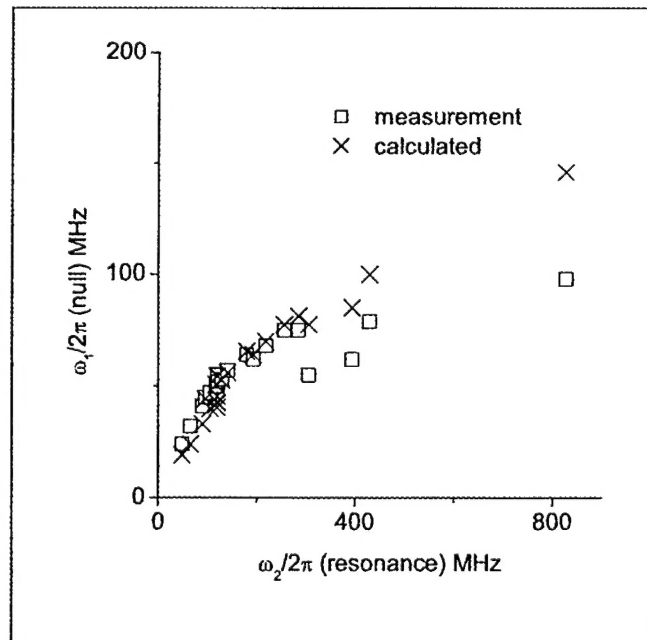


FIG. 8: The plasma sheath resonance frequency (ω_1) vs. the impedance resonance frequency (ω_2). The dashed line is the theoretical value inferred from probe measurements and assuming a sheath width of $s = 4.5\lambda_D$

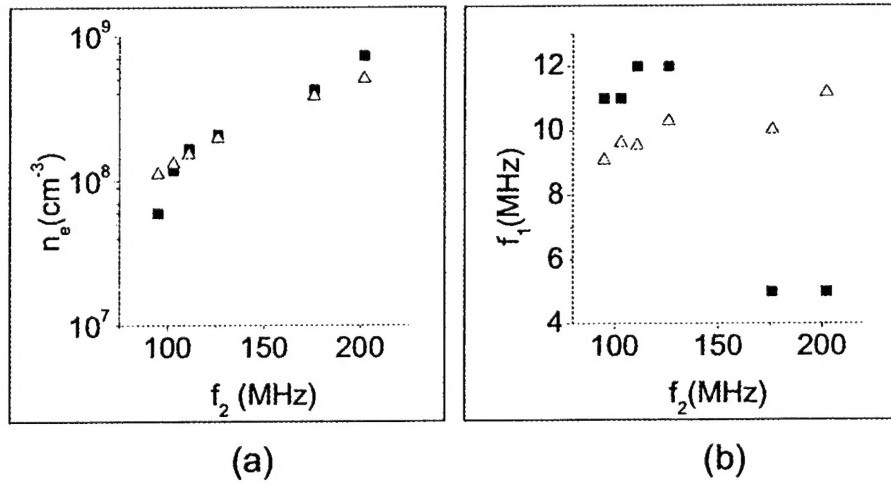


FIG. 9: Relationships between f_1 , f_2 , and n_e for a large (76 mm) diameter sphere.

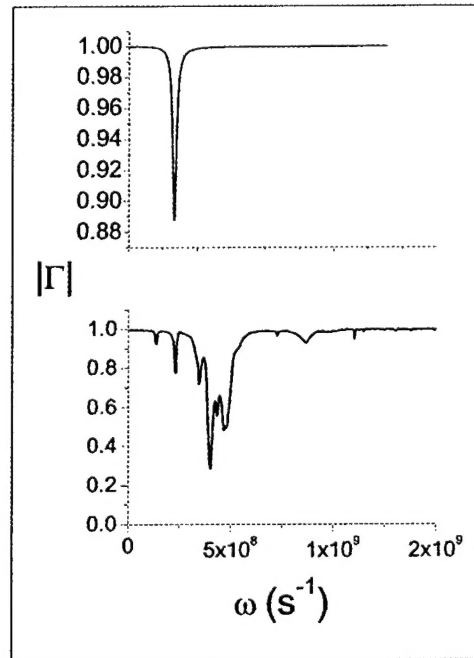


FIG. 10: Magnitude of the reflection coefficient vs frequency for the plasma conditions of figure 6. Top: Theoretical curve; bottom: measured curve.

IV. CONCLUSIONS

We have demonstrated the use of the plasma impedance probe at high frequencies using a commercial network analyzer. The probe is simple to construct and can be used remotely with a long transmission line length between the analyzer and the probe. It is ideal as a diagnostic in a plasma with large amounts of contaminants or reactive chemicals that would

interfere with the collecting surface of a Langmuir probe. In this regard it is similar to the plasma absorption probe built by Kokura [10]; however, the theory and data analysis is less complicated. The density indicated by the resonance in the impedance, given when $\omega = \omega_{pe}$, is on average within 25% of the density measured using a heated Langmuir probe. The location of the impedance null, referred to commonly as the plasma sheath resonance, agrees with the model proposed by Harp and Crawford [1]. This gives a sheath width of approximately 4.5 Debye lengths, which is also in agreement with earlier results. The large amount of energy deposited at this resonance indicates a collision frequency approximately an order of magnitude higher than theoretical estimates.

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